

OMAGIU LUI GRIGORE GHEBA (2)

Operații cu fracții algebrice – o introducere

1. Efectuați:

a) $\left(\frac{1}{a} + \frac{1}{b}\right) : \frac{1}{ab}$ R: $a + b$

b) $\left(\frac{1}{a} - \frac{1}{b}\right) \cdot \left(\frac{1}{a} + \frac{1}{b}\right) \cdot \frac{b^2 a^2}{b^2 - a^2}$ R: 1

c) $\frac{x+y}{x} + 2 + \frac{x-y}{y}$ R: $\frac{(x+y)^2}{xy}$

d) $\frac{x+y}{x^2 y} + \frac{x-y}{xy^2}$ R: $\frac{x^2 + y^2}{x^2 y^2}$

e) $\frac{1}{a^2} + \frac{2}{ab} + \frac{1}{b^2}$ R: $\frac{(a+b)^2}{a^2 b^2}$

f) $\frac{1}{25} - \frac{2}{5x} + \frac{1}{x^2}$ R: $\frac{(x-5)^2}{25x^2}$

g) $\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-a}{ca}$ R: 0

2. Calculați:

a) $\frac{x}{x+y} + \frac{2xy}{x^2 - y^2} + \frac{y}{x-y}$ R: $\frac{x+y}{x-y}$

b) $\left(\frac{1}{x-1} + \frac{1}{x+1}\right) : \left(\frac{1}{x+1} + \frac{2}{x^2-1} + \frac{1}{x-1}\right)$ R: $\frac{x}{x+1}$

c) $\frac{a}{a-b} - \frac{b}{a+b} - \frac{2ab}{a^2 - b^2}$ R: $\frac{a-b}{a+b}$

d) $\frac{a}{a+b} - \frac{b}{a-b} - \frac{a^2 + b^2}{a^2 - b^2}$ R: $\frac{2b}{b-a}$

e) $\left(\frac{3}{x-2} - \frac{2}{x+2} - \frac{10}{x^2-4}\right) \cdot \frac{x^2 - 4x + 4}{x}$ R: $\frac{x-2}{x+2}$

f) $\frac{4x^2 - 1}{8x} \cdot \left(\frac{2x+1}{2x-1} - \frac{2x-1}{2x+1}\right)$ R: 1

g) $\left(\frac{a+1}{a-1} + \frac{a-1}{a+1} - 1\right) : \left(\frac{4}{a^2-1} + 1\right)$ R: 1

h) $\left[1 - \left(\frac{1+b}{1-b} - 2 + \frac{1-b}{1+b}\right) : \frac{4b}{1-b}\right] : \frac{1}{1+b}$ R: 1

i) $\left(\frac{x-y}{x+y} + \frac{x+y}{x-y}\right) \cdot \left(\frac{x^2 + y^2}{2xy} + 1\right) \cdot \frac{xy}{x^2 + y^2}$ R: $\frac{x+y}{x-y}$

3. Calculați:

a) $\frac{18}{x^2-9} + \frac{x}{x+3} + \frac{3}{3-x}$ R: $\frac{x-3}{x+3}$

b) $\left(a - \frac{4ab}{a+b} + b\right) : \left(\frac{a}{a+b} - \frac{b}{b-a} - \frac{2ab}{a^2 - b^2}\right)$ R: $a - b$

c) $\left(\frac{x}{x^2-4} + \frac{2}{2-x} + \frac{1}{x+2}\right) : \left(x - 2 + \frac{10-x^2}{x+2}\right) \cdot (2-x)$ R: 1

d) $\left(\frac{3}{2x-1} + \frac{7}{2x+1} + \frac{20x-4}{1-4x^2}\right) : \frac{2}{x+1}$ R: 0

e) $\left(u - \frac{3uv}{u+v} + v\right) : \left(\frac{u}{u+v} - \frac{v}{v-u} - \frac{uv}{u^2 - v^2}\right)$ R: $u - v$

f) $a \cdot \left(\frac{1}{1-a} + \frac{1}{a+1}\right) : \left(\frac{1}{1-a} - \frac{1}{a+1}\right)$ R: 1

g) $\frac{1}{x+2} + \left(\frac{1}{2-x} + \frac{1}{x+2} - 1\right) : \frac{x^3 + 2x^2}{x^2 - 4x + 4}$ R: $\frac{4}{(x+2)^2}$